Fluidized Bed Freeboard Phenomena: Entrainment and Elutriation

A model describing the entrainment of solid particles in the freeboard region of a fluidized bed is proposed. Published experimental data have been used to correlate the entrainment rate near the bed surface and the elutriation rate above the bed. It has been shown that the column diameter can have an important influence on the amount of fines elutriated.

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SCOPE

When the fluidized bed is used as a chemical reactor the freeboard region not only provides the space for the disengagement of solids, but also for additional reaction between the projected particles and the gas. For fast reactions, this region could become very important providing considerable conversion of gas (Yates and Rowe, 1977).

In order to model physical and chemical phenomena in this region, it is necessary to understand the entrainment of particles in the freeboard. Extensive studies on entrainment and elutriation have been made, and many correlations have been proposed (Leva, 1951; Osberg and Charlesworth, 1951; Jolley and Stantan, 1952; Yagi and Aochi, 1955; Zenz and Weil, 1958; Andrews, 1960; Wen and Hashinger, 1960; Thomas et al., 1961; Lewis et al., 1962; Sanari and Kunii, 1962; Blyakher and Pavlov, 1966; Hanesian and Rankell, 1968; Tweddle et al., 1970; Guha et al., 1972; Tanaka et al., 1972; Fournol et al., 1973; Harrison et al., 1974; Merrick and Highley, 1974; Nazemi et al., 1974; Large et al., 1976; Bachovchin et al., 1979; Geldart et al., 1979; Colakyan et al., 1979; Lin et al., 1980). However, most of them are based on the experimental data of small-scale fluidized beds which are found to be much lower than that of large-scale fluidized beds. Extrapolation of these empirical correlations usually leads to strange results.

It is the objective of this study to correlate the entrainment and elutriation data of different column diameters and to propose a more realistic model for estimation of the entrainment rate of solid particles in the freeboard. In addition, where the projected particles originate and how the bed internals affect the entrainment and elutriation rates are discussed.

CONCLUSIONS AND SIGNIFICANCE

The solid particles inside the bed are splashed into the freeboard when bubbles burst at the bed surface. The particles that are thrown above the bed either rise or fall in this region, depending on the size, density and the gas velocity. The entrainment rate of particles decreases exponentially along the freeboard height. The rate of particles ejected at the bed surface can be correlated in terms of hydrodynamic parameters of the bed, such as bubble diameter and excess gas velocity above minimum fluidization. The elutriation rate of fines, on the other hand, is practically independent of the bed hydrodynamics, namely, bed particle size, presence or absence of baffles or tubes in the bed, etc. However, the installation of internals above the bed will reduce the entrainment and elutriation due to the direct impingment on the obstructions. The elutriation rate is related to the saturation carrying capacity of the gas stream, which can be calculated from the equations based on material and force balances. Both the entrainment rate and the elutriation rate are affected by the size of the column. The elutriation rate is especially affected by the wall of the column when the solid velocity is low. This is because the particles tend to descend along the wall of the column.

INTRODUCTION

In order to examine gas and solid flow patterns, a fluidized bed may be divided into three distinct zones as shown in Figure 1. Close to the bottom of the bed is the distributor zone or grid region which is represented by vertical or horizontal gas jets and/or small size gas bubbles. The type and the design of the distributor determine the form of gas voids. Above the distributor zone is the bubbling zone. In the bubbling zone, bubbles grow by coalescence and rise to the surface of the bed where they break. As bubbles break at the surface of the bed, particles are thrown up above the bed surface and are entrained by the upward flowing gas stream. This zone above the bed surface is the freeboard zone. In this zone some particles are carried far above the bed surface and are elutriated, while others fall back to the bed. The height at which the entrainment rate becomes nearly constant is often called the transport disengaging height (TDH). The freeboard zone affords an opportunity for the disengagement of particles and for the lean phase reactions. During the operation of a fluidized bed, a large amount of fine particles could be elutriated continuously. For example, in the operation of a fluidized bed coal combustor (FBC), the entrainment of carbon from the freeboard represents the main source of loss in combustion efficiency. Therefore, it may become necessary to recycle the entrained fines back into the fluidized bed or to install a carbon burn-up cell to increase the combustion efficiency. In addition, an excess amount of fines elutriated can heavily tax the performance of cyclones and bag filters. In an FBC operation, the extent of fines loading in the freeboard, both carbon and limestone, not only significantly affects the SO_2 absorption, NO_x reduction, CO and hydrocarbon emission, etc., but also contributes

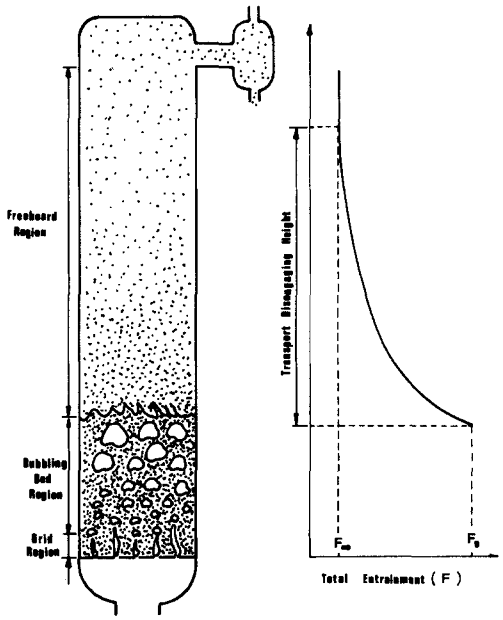

Figure 1. Particle entrainment from fluidized bed.

to other physical phenomena such as tube heat transfer and particle attrition.

Although a large number of correlations on rates of elutriation and entrainment are available in literature, many are unreliable or unsatisfactory for estimating entrainment rates in design and freeboard modeling. The lack of good correlations stems mainly from the difficulty in obtaining accurate entrainment rate data. Data obtained by various investigators using similar equipment and operating under similar conditions could differ considerably in some instances. Also, many of these experiments were performed using relatively small diameter beds. Data from large-scale beds are scarce and the effects of bed diameter and internals on entrainment which could be considerable have not been clearly established.

The purpose of this study is to provide a new perspective based on the existing entrainment and elutriation data and to propose a new correlation which is more reliable than existing correlations for the simulation and modeling of freeboard phenomena in fluidized beds.

HISTORY OF DEVELOPMENT

In early studies, the rate of elutriation of fines from fluidized beds was measured by Leva (1951) who expressed the rate by a first order equation:

$$-\frac{dX_i}{dt} = kX_i \tag{1}$$

Osberg and Charlesworth (1951) also used the first order expression and correlated the value of k in terms of variables such as the particle diameter of the bed, particle diameter of the fines, solid velocity and bed height at the incipient fluidization. Most of the studies that followed these pioneering works were devoted to the development of correlations for k, the elutriation velocity constant, that best fitted the experimental data.

Yagi and Aochi (1955) redefined the rate expression of elutriation as:

$$\frac{dX_i}{dt} = -E_{i\infty}\frac{A}{W}X_i \tag{2}$$

where, $E_{i\infty}$, the elutriation rate constant, is independent of bed height and is equal to kW/A.

Dimensionless groups such as $E_{i\infty}/\rho_g(U_o - U_{ts})$, $d_p U_{ts} \rho_g/\mu$, $(U_o - U_{ts})^2/g\, d_p$ and $(\rho_s - \rho_g)/\rho_g$ have been used to relate $E_{i\infty}$ with system parameters by Wen and Hashinger (1960). Tanaka et al. (1972) used the same groups to correlate mostly their own data.

Zenz and Weil (1958) introduced the concept of the saturated carrying capacity of solids in a pneumatic transport system and suggested a correlation employing $U_o^2/g\, d_p\, \rho_s^2$ and $E_{i\infty}/U_o\, \rho_g$ to estimate the elutriation rate constant. They also studied the entrainment of particles below the TDH.

Lewis et al. (1962) studied the effect of column diameter and the presence of internals in the dense bed on the entrainment and elutriation of particles. The effects of grid, screen packings and baffles in the freeboard on the entrainment rate were also studied by Blyakher and Pavlov (1966), Tweddle et al. (1970), and Harrison et al. (1974), respectively.

Instead of a first order equation used by other investigators, Thomas et al. (1961) used the expression:

$$\frac{dX_i}{dt} = -kX_i\,(1 + bX_i) \tag{3}$$

to represent their elutriation rate data. Another modified first order equation was proposed by Hanesian and Rankell (1968):

$$\frac{X_i}{X_{io}} = b_i\, e^{-k_1 t} + (1 - b_1)\, e^{-k_2 t} \tag{4}$$

Guha et al. (1972) attempted to develop a correlation on the rate of elutriation. Unfortunately, the data they used were apparently taken well below the TDH and gave considerably higher values than most of the investigators.

A relatively large-scale fluidized bed (0.6 m) was used by Bergougnou and his co-workers (Fournal et al., 1973; Nazemi et al., 1974; Large et al., 1976) to study the entrainment phenomena. An entrainment model was proposed which followed closely that described by Kunii and Levenspiel (1969):

$$F_i = F_{i\infty} + F_{io}\exp(-ah) \tag{5}$$

Merrick and Highley (1974) also collected elutriation rate data from a large-scale fluidized bed (0.90 × 0.45 m) using coal-ash particles. They employed the group $(U_o - U_{mf})$ to correlate the elutriation data. As will be indicated later the elutriation rate is believed to be unaffected by the bed hydrodynamics. Therefore, the term U_{mf} would not be a proper variable to use for correlation of the elutriation rate.

Recently, Geldart et al. (1979) performed elutriation experiments with a bed composed of sand and alumina particles. Colakyan et al. (1979) studied the elutriation of sand particles in a 0.9 × 0.9 m fluidized bed with and without heat transfer tubes and correlated their data with the group $(1 - U_{ts}/U_o)$. Bachovchin et al. (1979) also obtained the data on entrainment and elutriation at high gas velocities from a small bed (0.1524 m). A correlation that considers the bed stratification was proposed. Rates of entrainment and elutriation of char particles from a 0.6 × 0.6 m bed were reported by Lin et al. (1980).

A summary of experimental conditions used by the previous investigators in the studies of entrainment and elutriation is tabulated in Appendix 1. The published correlations on elutriation rate constants are listed in Appendix 2.

ENTRAINMENT AT THE BED SURFACE

Origin of Ejected Particles

The mechanism of solids ejection at the bubbling bed surface is still not well understood. The origin of ejected particles is reported to be primarily due to the following two sources:

(a) Some of the solids which have been lifted by the bubble wake are thrown upwards following the bubble burst at the bed surface (Leva and Wen, 1971; Yates and Rowe, 1977).

(b) Solids contained in the leading bulge of the bubble are detached at the surface when the bubble breaks (Do et al., 1972).

George and Grace (1978) performed experiments in a bed (1.0 m deep, 0.108 m dia.) of flint silica sand (210 ~ 500 μm). Thin layers of coke particle (53 ~ 149 μm) were put on the top of the bed. Ejected particles were then caught 3 cm above the bed surface. They observed that the fraction of coke particles collected was always less than 3% (by weight) of the total for the coke layers which varied from 0.072 to 0.197 cm. They concluded that the vast majority of ejected particles did not originate from the surface layers. However, Rowe and Partridge (1962) performed the following experiment and concluded that the mechanism (b) is the dominant one. They injected a bubble into a two-dimensional bed which had a layer of dyed solids above the gas distributor. The idea was to demonstrate the mixing and shedding of solids behind the rising bubble. Their results showed that the colored solids in the wake were retained in the bed after the bubble erupted from the bed surface. Do et al. (1972) used glass beads (177 ~ 250 μm) in a two-dimensional bed to perform the experiments. They filmed the motion of bubbles in the bed. The photographs showed that the ejected particles originated at the nose of the bursting bubbles and not from the bubble wakes.

In view of these experimental evidences, it can be concluded that the solids ejection is due to both mechanisms at this time.

Correlation of Entrainment Rate Constant at Bed Surface

By assuming the ejected particles originated from the bulge, Chen and Saxena (1978) derived equations for the solid projection rate at the surface. Their model was shown to predict the solid entrainment rate constant at the surface of a freely bubbling bed as a function of the excess gas velocity. However, data from a slugging bed obtained by Jolley and Stantan (1952) were used to fit the freely bubbling model. Hence their contention that the model fits the diverse data of freely bubbling beds was not verified. George and Grace (1978) correlated the volume of particles ejected into the freeboard as a function of the bubble diameter. In either case, the amount of particles being entrained near the surface of the bed is related to the bubbling phenomena regardless of the point of origin. Thus, it appears possible to correlate the entrainment rate of particles at the bed surface, F_o, with the bubble diameter, D_B, and the excess gas velocity, $(U_o - U_{mf})$. The entrainment rate at the bed surface can be obtained by extrapolation of the entrainment rate data in the freeboard to the bed surface. Such a correlation is presented in Figure 2. The correlation equation is given by:

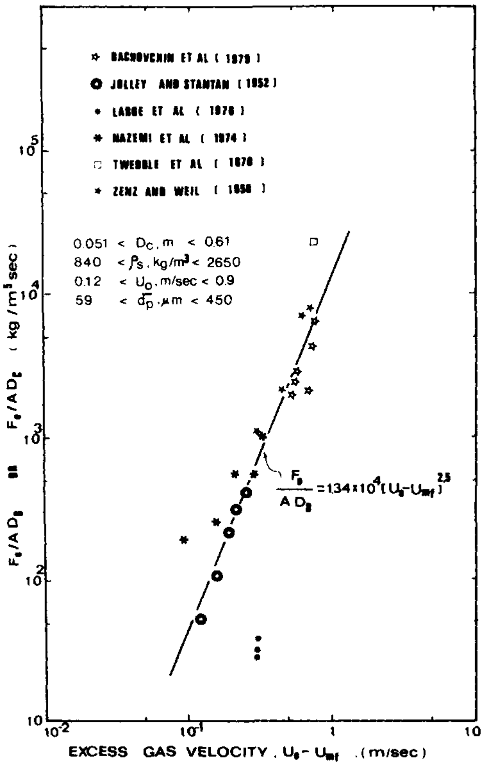

Figure 2. Correlation of entrainment rate data at the bed surface.

$$\frac{F_o}{A\,D_B} = 3.07 \times 10^{-9} \frac{\rho_g^{3.5}\, g^{0.5}}{\mu^{2.5}} (U_o - U_{mf})^{2.5}\ \mathrm{kg/m^5{\cdot}s} \tag{6}$$

where A is the cross-sectional area of the column. D_B is the bubble diameter at the bed surface and can be calculated using an appropriate equation at different flow regimes (Cranfield and Geldart, 1974; Mori and Wen, 1975). U_{mf}, the minimum fluidization velocity, is estimated by the equation of Wen and Yu (1966). For slugging beds, column diameter, D_c, is used instead of D_B. The amount of fine particles projected into the freeboard at the bed surface, F_{io}, depends on the weight fraction of that size of fine particles present in the bed, X_i, and can be approximated by an equation having a form similar to Henry's law as:

$$F_{io} = F_o \cdot X_i \tag{7}$$

ELUTRIATION OF FINES

Experimental evidence shows that only a small particle whose terminal velocity is less than the gas velocity will be carried out of the freeboard if the freeboard height is tall enough and if no internals are present in the freeboard. The amount of the particles elutriated, $F_{i\infty}$, is determined by both the elutriation rate constant, $E_{i\infty}$, and the weight fraction of the fine particles present in the bed, X_i, as:

$$F_{i\infty} = E_{i\infty} \cdot X_i \tag{8}$$

The total amount of particles elutriated can be calculated by the summation of the amount of each particle elutriated. Thus,

$$F_\infty = \sum_i F_{i\infty} \tag{9}$$

Alternately, the elutriation rate constant, $E_{i\infty}$, can be regarded as the total amount of particles blown away from a bed composed of one-sized particles. $E_{i\infty}$ is defined mathematically as:

$$-\frac{W}{A}\frac{dX_i}{dt} = E_{i\infty}\, X_i \tag{2}$$

Many correlations published (Appendix 2) for determining the elutriation rate constant are limited to the experimental conditions employed by each of the investigators (Yagi and Aochi, 1955; Zenz and Weil, 1958; Wen and Hashinger, 1960; Tanaka et al., 1972; Merrick and Highley, 1974; Bachovchin et al., 1979; Colakyan et al., 1979; Geldart et al., 1979). Extrapolation of these correlations to different operating conditions (different column diameters) often lead to very strange results (Matsen, 1979; Lin et al., 1980). A more reliable correlation was proposed recently (Wen and Chen, 1980). In this paper, this correlation is improved and refined to make it more accurate by testing with more experimental data.

When particles of a single size are elutriated above the TDH, the saturation carrying capacity of the gas stream under pneumatic transport conditions is reached (Zenz and Weil, 1958). The rate of elutriation from a bed composed of a single-sized particle can be viewed to be independent of a hydrodynamics prevailing inside the bed. A material balance of elutriating particles in such a condition can be expressed as:

$$E_{i\infty} = \rho_s\,(1 - \epsilon_i)\, U_{si} \tag{10}$$

where, ρ_s is the solid density of elutriated particles, and ϵ_i is the voidage in the freeboard when the single-sized particles are being elutriated. U_{si}, the solid velocity of a given particle size, can be approximated by the difference between the gas velocity and the single particle terminal velocity as $(U_o - U_{ts})$. Leung et al. (1971) calculated the solid flow rate at the onset of choking in a vertical pneumatic transport system by the same equation and assumed the voidage was equal to 0.97. However, the voidage in the freeboard, ϵ_i, can be obtained from the force balance on the wall (Wen, 1971; Yang, 1975; Wen and O'Brien, 1976) as:

$$U_{si} = U_o - U_{ts}\sqrt{\left(1 + \frac{\lambda \cdot U_{si}^2}{2g\,D_c}\right)\epsilon_i^{4.7}} \tag{11}$$

Substituting the approximated solid velocity, U_{si}, with $(U_o -$

U_{ts}), Eq. 11 may be rearranged to obtain ϵ_t, as:

$$\epsilon_t = \left[1 + \frac{\lambda(U_o - U_{ts})^2}{2g\,D_c}\right]^{-1/4.7} \tag{12}$$

λ, the friction coefficient due to the bouncing of the particles against the wall and against each other is defined as:

$$dF_f = \pi D_c\,dh \cdot \tau_s = \frac{\lambda\,U_{si}^2}{2g_c\,D_c} \cdot dW_s = \frac{\lambda\,U_{si}\,G_s}{2g_c\,D_c}\,dh \tag{13}$$

A constant value of λ (equal to 0.01) was proposed by Yang (1975) in comparison with the experimental data obtained from vertical pneumatic transports. Further work has been carried out in this paper to correlate λ with the system parameters, as:

$$\frac{\lambda\,\rho_s}{d_p^2}\left(\frac{\mu}{\rho_g}\right)^{2.5} = \begin{cases} 5.17\,Rep^{-1.5} \cdot D_c^2 & \text{for } Rep \le Rep_c \quad (14) \\ 12.3\,Rep^{-2.5} \cdot D_c & \text{for } Rep \ge Rep_c \quad (15) \end{cases}$$

where,

$$Rep_c = 2.38/D_c \tag{16}$$

and

$$Rep = \rho_g\,(U_o - U_{ts}) \cdot dp/\mu \tag{17}$$

Figure 3 shows the comparison of this relation with the experimental data (Leva, 1951; Yagi and Aochi, 1955; Andrews, 1960; Wen and Hashinger, 1960; Lewis et al., 1962; Tanaka et al., 1972; Merrick and Highley, 1974; Colakyan et al., 1979; Lin et al., 1980). The comparison of the calculated rate constant based on this correlation with the experimental data is shown in Figure 4. 80% of the experimental data can be represented by this correlation to be within ±50% deviation. It is noted that the deviation of some of the experimental data is greater than ±50%. In view of the diversity of the experimental conditions and the difficulty in obtaining accurate elutriation data, the deviation of this magnitude should be considered within experimental accuracy.

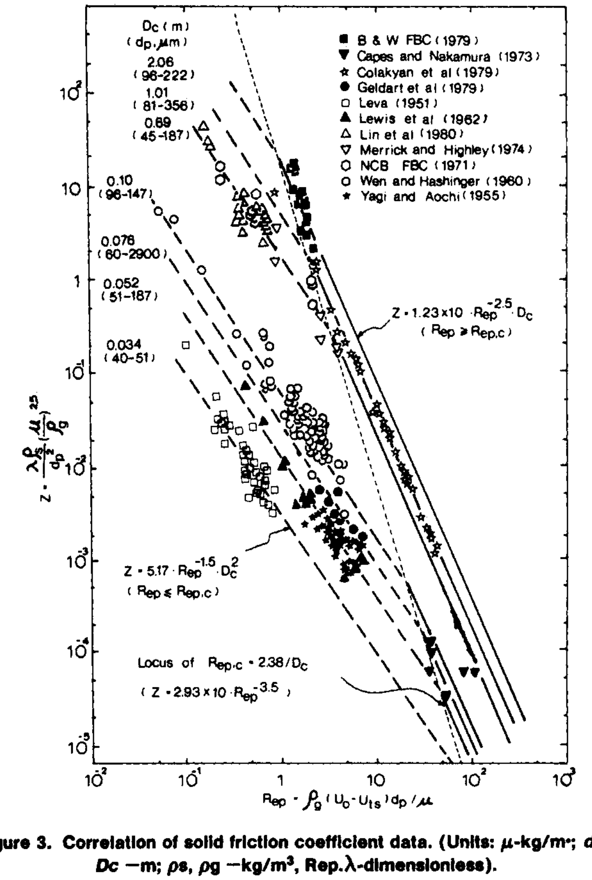

Figure 3. Correlation of solid friction coefficient data. (Units: μ-kg/m·s; dp, Dc —m; ρs, ρg —kg/m³, Rep.λ-dimensionless).

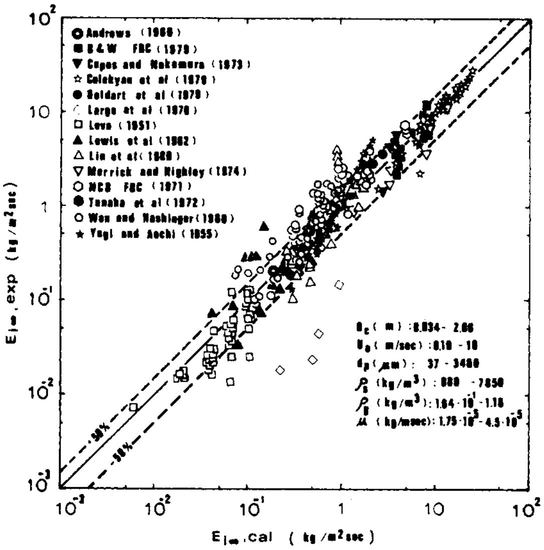

Figure 4. Comparison of elutriation rate constant data with the calculated values showing accuracy of the correlation.

ENTRAINMENT MODEL

In the freeboard region, solid particles either rise or fall back, depending on the size of the particles and the gas velocity. By assuming that the energy distribution of particles at the bed surface follows the Maxwell-Boltzmann distribution law, Andrews (1960) was able to show an exponential decay of the entrainment rate with respect to the freeboard height. Kunii and Levenspiel (1969) also explained the exponentially decaying rate based on the solid interchange between three distinct phases: upward-moving dispersed solids corresponding to a well established pneumatic transport regime and the ascending and descending particle agglomerates, corresponding to the particles moving upward and downward above the bed. The exponential decrease of the entrainment rate with respect to the freeboard height above the bed was also reported to result from: (a) the exponential dissipation of the effective gas velocity with freeboard height (Zenz and Weil, 1958); and (b) the solid particles being ejected from the bed surface into the freeboard above, which follow a certain velocity distribution (George and Grace, 1978).

Thus, the entrainment rate of solids in the freeboard can be represented by the following equation which is similar to that proposed by Large et al. (1976):

$$F = F_\infty + (F_o - F_\infty) \cdot \exp(-ah) \tag{18}$$

where, F is the entrainment rate of particles in the freeboard at a height, h, above the bed surface, and F_∞ is the elutriation rate (or the entrainment rate above the TDH). Here, the upper limit of TDH is a gradual and not a sharp demarcation line. In this paper the word TDH is used in the loose sense as conventionally used: "the height of freeboard above which rate of solids entrainment does not change appreciably." F_o is the entrainment rate at the bed surface and a is a constant representing the characteristics of the fluidized bed entrainment system.

For large particles, the entrainment rate above the TDH, F_∞, is negligibly small compared to the entrainment rate at the bed surface, F_o. Thus, Eq. 18 can be simplified as:

$$F = F_o \cdot \exp(-ah) \tag{19}$$

On the other hand, for small particles the entrainment rate, F, is approximately equal to the elutriation rate above the TDH, F_∞, thus,

$$F = F_\infty \tag{20}$$

Figure 5 shows the experimental data reported by Large et al. (1976) using silica sand in a 0.6 m fluidized bed. For particles less

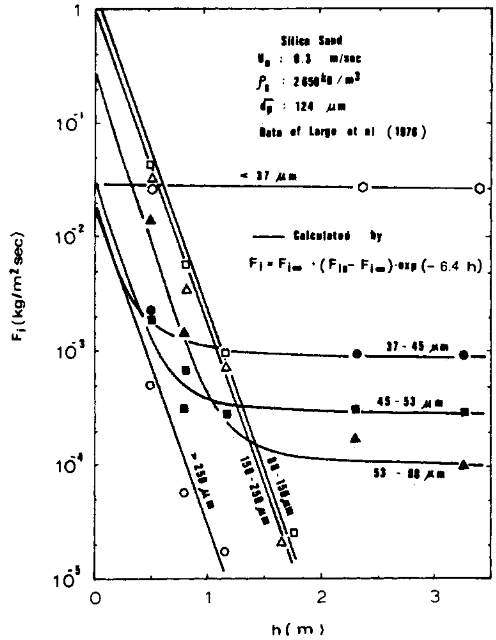

Figure 5. Comparison of entrainment rate data of Large et al. (1976) with the proposed model. (Calculated values for F_{io} and $F_{i\infty}$ were chosen to fit the data.)

than 37 μm the entrainment rate follows Eq. 20, while for particles larger than 88 μm, the entrainment rate lines follow Eq. 19.

The entrainment data available in the literature (Jolley and Stantan, 1952; Zenz and Weil, 1958; Tweddle et al., 1970; Nazemi et al., 1974; Large et al., 1976; Bachovchin et al., 1979) are correlated based on Eq. 18 by calculating F_o and F_∞ from the equations discussed. Figures 6 and 7 show comparisons of the experimental

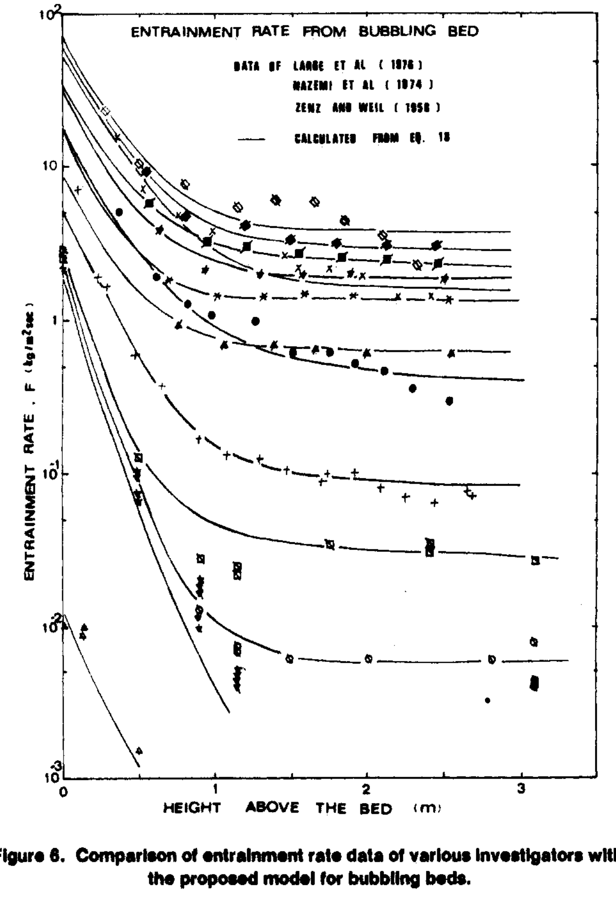

Figure 6. Comparison of entrainment rate data of various investigators with the proposed model for bubbling beds.

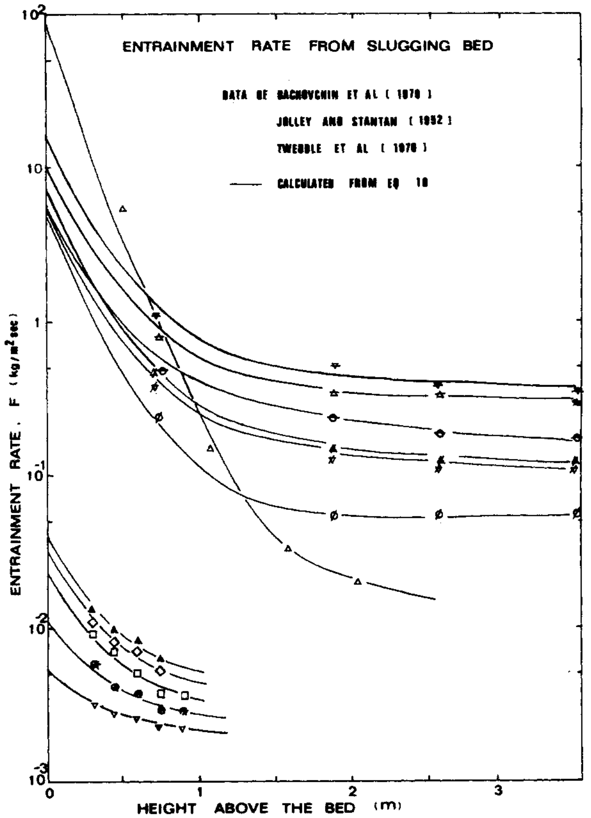

Figure 7. Comparison of entrainment rate data of various investigators with the proposed model for slugging beds.

data with the proposed correlations plotted on a semilog scale for bubbling and slugging beds, respectively. The value of "a" can be obtained from the slope of the straight line portion of this curve. As shown in Table 1, the value of a is not a strong function of the bed composition or gas velocity. Therefore, an average value of a is evaluated for each case. This correlation with a as a constant appears to agree well with the experimental data. Deviation from the experimental data is less than ±50% for 95% of the data examined as shown in Figure 8.

DISCUSSION

Entrainment Model

Freeboard Height. The entrainment model proposed here is based on the assumption that the entrainment rate of particles decreases exponentially along the freeboard. The value of TDH is not required in estimating the amount of solids being entrained. The amount of solids entrained at different heights in the freeboard, F, can be estimated from the entrainment rate at the bed surface, F_o, and from the elutriation rate, F_∞. The entrainment rate at the bed surface is calculated by knowing the bubble size at the bed surface. The elutriation rate, on the other hand, is estimated from an elutriation rate constant correlation which is related to the saturation carrying capacity of the gas stream but is independent of the bed hydrodynamics. The freeboard height required can be estimated from Eq. 18 by assuming that the entrainment rate at the freeboard outlet is within a small percentage more than that of the amount of particles elutriated. For example, if the entrainment rate is within one more percent of elutriation rate, the freeboard height required is estimated to be: TDH = $(1/a)\ ln\ (F_o - F_\infty)/(0.01\ F_\infty)$. Tanaka and Shinohara (1978) suggested that one percent of the total amount of particles entrained at the bed surface be used for the calculation of TDH, i.e., $F(\text{TDH}) = 0.01\ F_o$. This approximation is not suitable for calculation of TDH because the value so obtained is too small compared to experimental data.

TABLE 1. LEGEND TO FIGURES 6 AND 7

Symbol	Investigator	$\bar{d}p$ (μm)	ρ_s (kg/m^3)	D_c (m)	U_0 (m/s)	a (L/m)	a (average)
Ø	Bachovchin et	450	2630	0.1524	0.73	4.7	
▽̸	al. (1979)	448				3.5	
△̸		445				4.0	
○		450			0.90	4.0	4.0
⊖△		448				3.8	
⊖▽		445				3.8	
▽	Jolley and	76	1330	0.0508	0.1219	2.2	
⊗	Stantan (1952)				0.1524	3.0	
□					0.1905	3.7	3.5
◇					0.2240	4.2	
▲					0.2500	4.4	
▽̸	Large et al.	136	2650	0.61	0.30	6.4	
□̸	(1976)	124			0.30	6.4	
⊘		123			0.30	6.6	6.4
▽̸		123			0.20	6.4	
▲	Nazemi et al.	59	840	0.61	0.0914	3.5	
★	(1974)				0.1524	3.3	
▼					0.2134	3.7	3.6
■					0.2743	3.6	
◆					0.3353	3.9	
△	Tweddle et al. (1970)	163	1370	0.1651	0.762	6.1	6.1
+	Zenz & Weil	60	940	0.0508 × 0.61	0.3048	5.0	
●	(1958)				0.4572	3.6	
X					0.6096	4.2	4.2
◇̸					0.7163	4.1	

Constant a. The value of a in the entrainment equation (Eq. 18) varies from 3.5 to 6.4 m^{-1} as shown in Table 1. Since the value is not very sensitive in the estimation of the entrainment rate, it is recommended that a value of 4.0 m^{-1} be used for a system in which no information on entrainment rate is available.

Entrainment at Bed Surface

Experimentally, the entrainment rate at the dense bed surface is not easy to measure and fluctuates significantly even under a specific operating condition. Thus, F_o in the correlation (Eq. 6), is an average value of the entrainment rate at the bed surface. Its application to a range beyond that specified is not recommended.

Origin of Ejected Particles. Although it is known that particles ejected into the freeboard are due to bubbles erupting at the surface, the origin of ejected particles remains unclarified. It was observed in the slugging bed that the particles are lifted up by the slug and splashed into the freeboard when it breaks the surface.

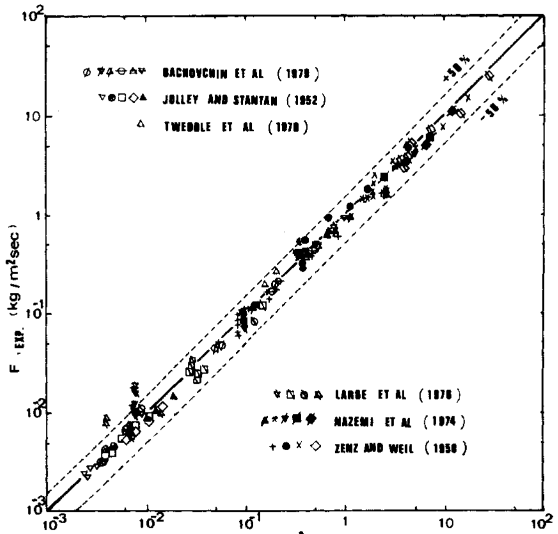

Figure 8. Comparison of entrainment rate data with the calculated values showing accuracy of the model.

However, the particles in the bubbling bed are lifted up by the bubbles. When the bubbles burst at the surface, the upward-moving momentum of the particles will carry them into the freeboard. Therefore, the origin of the projected particles may be resolved by the following postulation:

"The projected particles originate mainly from the wake of the bubbles erupting in the bubbling bed; whereas, in the slugging bed, the ejected particles come from the bulge of the slug." Observations that can support the postulation are made by George and Grace (1978) in a three-dimensional bubbling bed and by Rowe and Partridge (1962) and Do et al. (1972) in two-dimensional beds where rising bubbles are similar to slugs because of the wall effect. However, further study is needed to elucidate the mechanism ejection and the origin of ejected particles.

Effect of Bed Hydrodynamics. The value of F_o depends on the bubble diameter, column diameter and the excess gas velocity. Any changes inside the bed which affect the hydrodynamics (distributor plate, material composition or internals) will cause the amount of particles entrained at the bed surface and in the freeboard zone below TDH to change. Entrainment experiments performed by Lewis et al. (1962) in the 0.075 m bed with a 1.12 m freeboard height showed that stirrers (shaft diameter = 0.008 m) in the bed cut the entrainment considerably to a fraction of the norm (1/2 to 1/5) at high gas velocities (> 0.4 m/s) when the freeboard height was less than the TDH.

Elutriation of Fines

Effect of Bed Hydrodynamics. It is believed that the elutriation of fines is affected by the superficial velocity of gas but not affected by the bed hydrodynamics. That is, the presence of tubes or modes of operation have no effect on the elutriation rate. Sanari and Kunii (1962) reported that changing the coarse particle size (from 300 to 2500 μm) has no appreciable effect on the elutriation of fines (125 ~ 177 μm). Recently, Lin et al. (1980) entrained char particles from the bed of sand particles (125 ~ 419 μm) in a 0.6 × 0.6 m rectangular fluidized bed using two different methods. In one experiment, a thin layer of coke particles was put on the top of the bed before being elutriated while in the other experiment, fine char particles were mixed with the bed of sand particles. The elutriation rate from both cases were found to be nearly equal. Tanaka et al. (1971) also measured the rates of elutriation of glass beads from a 0.067 m bed by using porous and perforated distributor plates and found no difference in the elutriation rate constants. Thus, it can be concluded that the elutriation rate is affected very strongly by the gas velocity but the hydrodynamics of bed which is characterized by variables such as minimum fluidization velocity, bubble size, bed internals etc. does not have any effect. The ma-

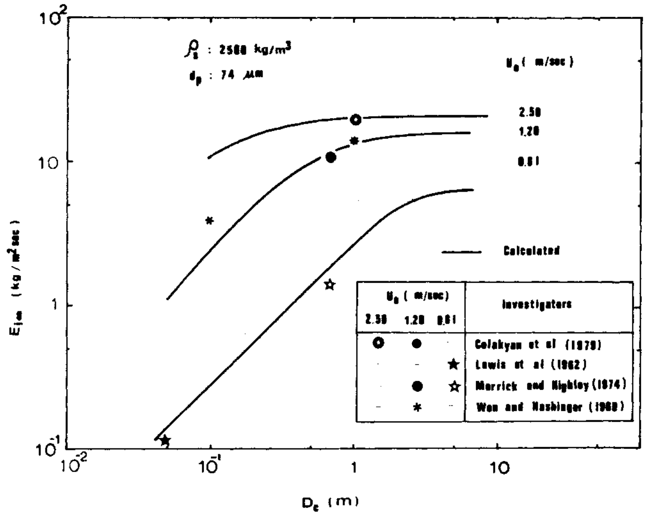

Figure 9. Effect of column diameter on elutriation rate constant.

jority of the elutriation rate constant correlations contain variables that are not related to the bed hydrodynamics. (Yagi and Aochi, 1955; Zenz and Weil, 1958; Wen and Hashinger, 1960; Tanaka et al., 1972; Colakyan et al., 1979; Geldart et al., 1979). However, the correlation proposed by Merrick and Highley (1974) indicates that minimum fluidization velocity of the bed particles have an effect on the elutriation rate constant. Further study indicates that the elutriation rate constants obtained by Merrick and Highley (1974) give a higher value especially at higher gas velocities (> 1.8 m/s). This suggests that these data may have been obtained below the TDH (fluidizing gas velocity: 0.6 to 2.4 m/s; freeboard height: 2.1 to 3.3 m). As a result, most of the data they obtained are for the entrainment rate rather than elutriation rate. The entraiment rate certainly, would be dependent on the bed hydrodynamics.

Effect of Column Diameter. Lewis et al. (1962) performed entrainment experiments using column diameter ranging from 0.019 to 0.146 m and concluded that for column diameters greater than 0.08 to 0.1 m, the entrainment or elutriation becomes constant and independent of vessel diameter; however, a sharp rise in entrainment rates was found at small bed diameters (0.009 to 0.02 m). Recently, more data from large-scale fluidized bed have been reported (Fournol et al., 1973; Nazemi et al., 1974; Merrick and Highley, 1974; Large et al., 1976; Colakyan et al., 1979; Lin et al., 1980). It is found that under similar operating conditions, the elutriation rate constant from the large-scale bed tends to be much higher than that from the small-scale bed as shown in Figure 9. At a low gas velocity (particle Reynolds Number less than the critical Reynolds Number), the friction between the particles and the column wall is important. Thus, a significant wall effect is observed. However, at a high gas velocity (particle Reynolds Number higher than the critical Reynolds Number), the friction between the particles themselves is more important, and the column diameter effect diminishes. But in any case, the descending fine particles appear to fall down predominantly along the wall (Wen and Hashinger, 1960; Horio et al., 1980). So when the column diameter becomes larger, a smaller fraction of fine particles will fall down. This means more are expected to be elutriated for larger columns.

Effect of Internals. As aforementioned, bed hydrodynamics has little, if any, effect on the elutriation rate constant. Therefore, the presence of internals inside the bed will not affect the amount of particles elutriated. Colakyan et al. (1979) measured the rates of elutriation of sand particles from a 0.9 × 0.9 m bed by putting the immersed tubes in different positions and reported that the presence or the position of a tube bundle in the bed does not affect the elutriation rates. However, the presence of internals above the dense bed will reduce not only the entrainment rate below the TDH but the elutriation rate above the TDH. This is because of the additional particle disengagement by direct impingement on

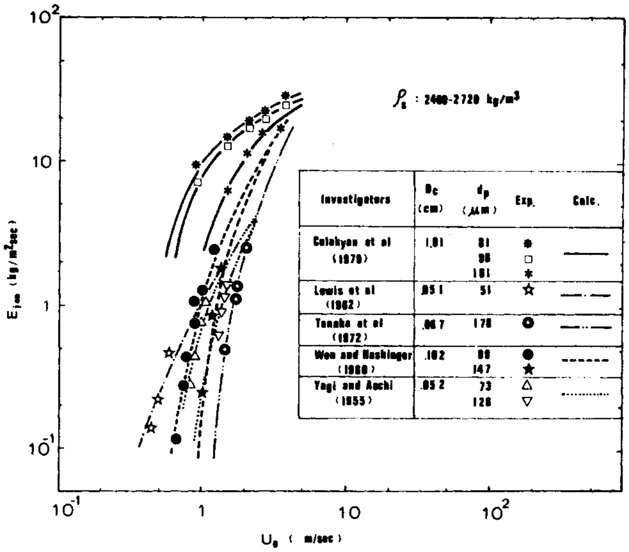

Figure 10. Effect of superficial gas velocity and particle size on elutriation rate constant for particles with density 2400 ~ 2720 kg/m³.

the obstructions in the freeboard and the particle flux drops below saturation carrying capacity. Experiments performed by Blyakher and Pavlov (1966), Tweddle et al. (1970) and Harrison et al. (1974) using grids, screen packings and baffles, respectively, above the dense bed showed that the rates of entrainment and elutriation are reduced dramatically. The rate of elutriation will not be affected if the internals in the lower part of the freeboard region can not reduce the flux down below the saturation carrying capacity (George and Grace, 1980).

Effect of Gas Velocity. The change of gas velocity will affect the elutriation rate constant tremendously. Leva (1951) reported the change of elutriation rate constant is roughly proportional to the 4th power of the gas velocity. The calculated values indicate that it is even greater and could be as high as the 6th power of the gas velocity for lower gas velocities. At higher gas velocities, the power diminishes to less than one. Figures 10 and 11 show the comparison of experimental data with calculated values for different particle size, column diameter, and particle density.

Effect of Particle Size and Particle Density. The effect of particle size on the elutriation rate constant is shown in Figure 12. It is observed in this Figure that the elutriation rate constant will reach a constant value for small particles and approach zero for

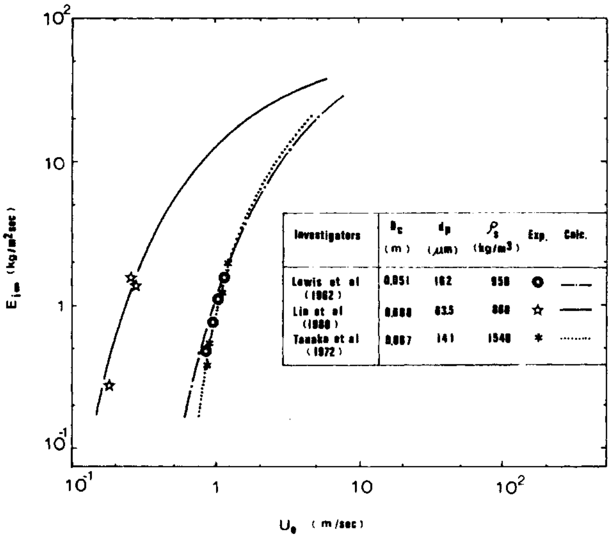

Figure 11. Effect of superficial gas velocity particle size and solid particle density on elutriation rate constant.

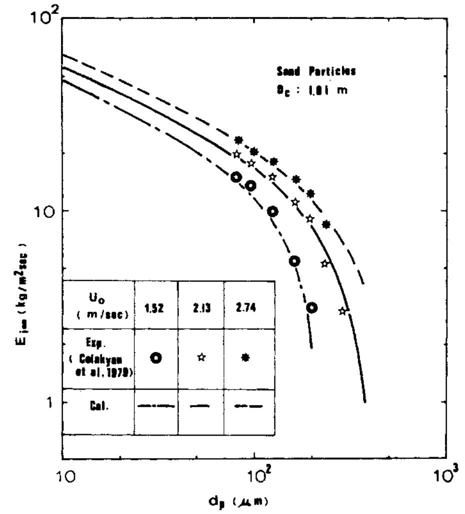

Figure 12. Effect of size of fines on elutriation rate constant for sand particles.

particles whose terminal velocity is larger than gas velocity. However, in some experimental works (Merrick and Highley, 1974; Geldart et al., 1979), a small amount of large particles with their terminal velocities greater than the gas velocity have been collected in cyclones. It is speculated that these observations result from either: (1) the height of the freeboard is not sufficiently high enough to allow coarse particles to return to the bed; or (2) the chanelling of the gas may have created zones of locally high velocity. The effect of particle density on the elutriation rate constant is shown in Figure 13. Although the equations used to calculate $E_{i\infty}$ do not illustrate explicitly the inverse relation with ρ_s, it does give a trend showing heavier particles have smaller elutriation rate constant.

Range of Applicability. Since the value of ϵ_i, the voidage of freeboard above the TDH for the bed with single-sized particle, ranges from 0.99 to 0.9999, a high accuracy of this number is required and the five digits after the decimal point is needed to assure the satisfactory estimation of elutriation rate constant. Because the

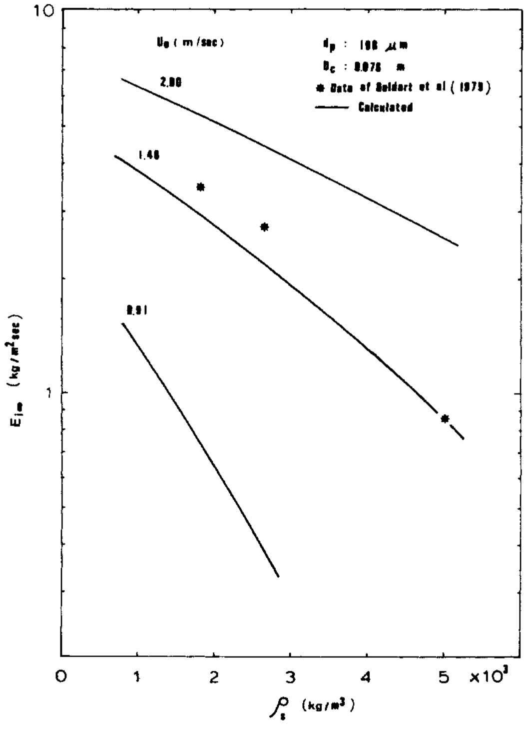

Figure 13. Effect of particle density on elutriation rate constant.

TABLE 2. COMPARISON OF ELUTRIATION DATA OF GEORGE AND GRACE (1980) D_c = 0.254 × 0.432m (EQUIVALENT COLUMN DIAMETER ASSUMED = 0.374m)

dpi (μm)	$E_{i\infty}$, exp. (kg/m²·s)	$E_{i\infty}$, cal (kg/m²·s)
	U_o = 0.99 m/s	
30	5.0	6.72
65.5	6.35	7.05
63.5	13.6	6.01
89.5	6.91	4.11
	U_o = 1.22 m/s	
30	14.5	14.0
65.5	13.0	10.2
63.5	5.7	9.38
89.5	3.53	7.55

data employed to correlate the elutriation rate constants are based primarily on experiments utilizing air at room temperature [except the data of Wen and Hashinger (1960) using Helium as fluidizing gas and that of B & W (1979) and NCB (1971) from FBC], the application of the correlation to other gases at different temperatures should be done very carefully. The range of applicability should not exceed the following:

$$
\begin{array}{rll}
0.1 & < U_o \ (\mathrm{m/s}) & < 10 \\
860 & < \rho_s \ (\mathrm{kg/m^3}) & < 7850 \\
37 & < d_p \ (\mu\mathrm{m}) & < 3400 \\
1.64 \times 10^{-1} & < \rho_g \ (\mathrm{kg/m^3}) & < 1.18 \\
1.75 \times 10^{-5} & < \mu \ (\mathrm{kg/m{\cdot}s}) & < 4.5 \times 10^{-5} \\
0.034 & < D_c \ (\mathrm{m}) & < 2.06
\end{array}
$$

Two Powder Beds. It is noted that the above discussion is based on the results from one powder bed (one kind of particle with different sizes). If more than two kinds of powders are used in the bed, the conclusion will not be the same, unless interaction or static force between the different kinds of particles is absent. Geldart et al. (1979) reported that the elutriation rate of sand particles was increased significantly when mixed with an equal quantity of alumina particles, indicating a strong interaction between this two kinds of particles. In these cases, the size of sand particles had a spread from 0 to 355 μm and alumina particles from 0 to 125 μm.

Comparison of Elutriation Correlations. The elutriation rate constant correlation proposed in this study is compared with those previously proposed by Yagi and Aochi (1955), Zenz and Weil (1958), Wen and Hashinger (1960), Tanaka et al. (1972), Geldart et al. (1979), Colakyan et al. (1979) and Lin et al. (1980). The experimental data shown in Figure 4 are used to test the accuracy of the correlations. The accuracy is expressed as mean deviation from the experimental data in percent and is defined as:

$$
\text{Mean Deviation, \%} = \left[\sqrt{\sum\left(\frac{E_{i\infty}\,\mathrm{cal} - E_{i\infty}\,\mathrm{exp}}{E_{i\infty}\mathrm{exp}}\right)^2} \Big/ N\right] \times 100
$$

The deviation for the previous correlations ranges from 40% to 1800% as compared to 20% of the present correlation for 170 data points. Two factors explain the poor agreement shown by all the previous correlations. First, the data used for the correlations were obtained under very limited operating conditions; therefore, extrapolation to the conditions beyond the range can result in extremely poor agreement. Secondly, the geometry and the size of the vessel can considerably affect the elutriation rate as a result of the wall effect. Most of the data used for the correlations were obtained from relatively small vessels. Thus, the present correlation appears to represent data from a wide variety of experimental conditions, using different sized fluidized beds. Unlike previous correlations, the proposed correlation can predict elutriation rate constants of different column diameters.

Recently, George and Grace (1980) performed the elutriation experiments in a rectangular (0.254 × 0.432 m) bed. Table 2 shows a comparison of the experimental data of George and Grace (1980) with the calculated values from the present correlation at two selected velocities of 0.99 and 1.22 m/s. At the high gas velocity, the comparison is fair; however, it is not satisfactory at the low gas velocity. This may be due to the non-circular bed. The solid friction coefficient (λ) in a rectangular (0.254 × 0.632 m) bed could be considerably different from that in the circular or square. Further study is needed to elucidate the effect of bed geometry and of freeboard flow pattern on the elutriation rate constant.

Reliability of Experiment Data. The entrainment data obtained using isokinetic probes tend to give lower values than those using funnels or cyclones to collect the entrained solids. This fact has been frequently observed by the previous investigators (Bergougnou, 1980; Zenz, 1980). The reason for this problem is that it is very difficult to adjust the pressure and the flow pattern inside of an isokinetic probe to be close to the conditions existing in the freeboard. The data obtained by Large et al. (1976) using isokinetic probes are considerably lower than that of other investigators who experimented at equivalent operating conditions. Therefore, these data have been eliminated in the correlation of entrainment rate at the bed surface shown in Figure 2. The isokinetic probe data of Large et al. (1976) for the elutriation rate are again found to be considerably lower than other investigators as shown in Figure 4.

ACKNOWLEDGMENT

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NOTATION

a	= constant in the entrainment equation, m^{-1}
A	= cross section area of the bed, m^2
b, b_1	= constant in Eqs. 3 and 4, –
d_p or d_{pi}	= particle diameter of close cut particle i, m
$\bar{d}_p$	= average particle diameter in the bed, μm
D_B	= bubble diameter at the bed surface, m
D_c	= column diameter, m
$E_{i\infty}$	= elutriation rate constant of particle size i, kg/m^2·s
F	= total elutriation rate of the particles, kg/m^2·s
F_f	= frictional force of the particles, kg-force
F_i	= entrainment rate of particle size i, kg/m^2·s
$F_{i\infty}$	= elutriation rate of particle size i, kg/m^2·s
F_o	= total entrainment rate at the bed surface, kg/m^2·s
F_∞	= total elutriation rate of the particles, kg/m^2·s
g	= gravitational acceleration constant, m/s^2
g_c	= gravitational conversion constant, m·kg/s^{2}kg-force
G_s	= solids flow rate, kg/s
h	= height above the dense bed surface, m
k, k_1, k_2	= elutriation velocity constant, s^{-1}
N	= number of experimental points, –
Rep	= particle Reynolds Number = $\rho_g (U_o - U_{ts}) dp/\mu$, –
Repc	= critical particle Reynolds Number, –
Ret	= dp $U_{ts}\rho g/\mu$,
t	= time, s
T	= temperature, °K
U_{mf}	= minimum fluidization gas velocity, m/s
U_o	= superficial gas velocity, m/s
U_{si}	= solid velocity (upward), m/s
U_{ts}	= single particle terminal velocity of particle size i, m/s
W	= weight fraction of the bed, kg
W_s	= weight of the solid particles in a vertical pipe having a length h, kg
X_i	= weight fraction of particle size i in the bed, –
X_{io}	= initial weight fraction of particle size i in the bed, –
ϵ	= voidage in the freeboard, –
ϵ_i	= voidage in the freeboard for the system having only particle size i, –
λ	= solid friction coefficient, –
ρ_g	= gas density, kg/m^3
ρ_s	= solid density, kg/m^3
μ	= viscosity of gas, kg/m·s
τ_s	= shear stress between solids and the wall, kg-force/m^2

APPENDIX 1.

Experimental Conditions for Entrainment or Elutriation of Fines from Fluidized Beds

Investigator	Experiments	Column Diameter, D_c Bed Height, L Freeboard, H (m)	Gas	Distributor	Particles	Size of Fines to be Entrained (μm)	Size of Coarse Solids in Parent Bed (μm)	Initial Weight Fraction of Fines (%)	Superficial Gas Velocity (m/s)	Internals
Leva (1951)	elutriation, two components, batch operation	$D_c = 0.035$ $L + H = 1.22$	air	perforated plate	sand and iron catalyst	41–67	110–230	20	0.21–0.41	none
Osberg and Charlesworth (1951)	elutriation, two components, batch operation	$D_c = 0.076$ $L = 0.14$–0.29	air, CO_2 H_e, Freon-12	sintered steel plate	glass spheres	19–36	590–1450	0.4–5.6	0.04–0.18	none
Jolley and Stantan (1952)	entrainment, one component, batch operation	$D_c = 0.0508$ $L = 0.648$ $H = 0.305$–0.914	air, H_2	—	pulverized coal	76	—	—	0.049–0.259	none
Yagi and Aochi (1955)	elutriation, two and multicomponents, batch operations	$D_c = 0.052$–0.071 $L = 0.031$–0.12	air	fixed bed of steel balls	sand, glass, seed, refactory	85–500	310–1640	5–20	0.92–1.62	none

Investigator	Experiments	Column Diameter, D_c Bed Height, L Freeboard, H (m)	Gas	Distributor	Particles	Size of Fines to be Entrained (μm)	Size of Coarse Solids in Parent Bed (μm)	Initial Weight Fraction of Fines (%)	Superficial Gas Velocity (m/s)	Internals
Zenz and Weil (1958)	entrainment of FCC catalyst, steady state operation	$D_c = 0.051 \times 0.61$ $H = 0.254$–2.80	air, mean pressure 8 atm	grid	FCC catalyst with size distribution	20–150	—	—	0.30–0.72	none
Andrews (1960)	entrainment, one component, batch operation	$D_c = 0.0965, 0.0305$ $H = 0.305$–3.96	air	—	3A catalyst	50, 60	—	—	0.518–0.368	none
Wen and Hashinger (1960)	elutriation, two and multicomponents, batch operation	$D_c = 0.051, 0.102$ $L + H = 2.08, 1.83$	air, He	filtercloth	glass spheres, coal powder	40–140	100–280	6–100	0.22–1.32	none
Thomas et al. (1961)	elutriation, two components, batch operation	$D_c = 0.102$ $L = 0.15$–0.92 $L + H = 1.83$	air	sintered bronze plate	glass beads	60–79	80–300	2–20	0.06–0.38	none
Lewis et al. (1962)	entrainment, one component, steady state operation	$D_c = 0.019$–0.146 $L = 0.102$–0.71 $H = 0.92$–3.16	air	wire mesh screen on fixed bed	glass, polystyrene, iron, cracking catalyst	51–361	—	—	0.28–4.76	with and without stirrer or wire obstruction
Blyakher and Pavlov (1966)	entrainment, one component, batch operation in conical vessels	upper D_c/lower D_c = 15/6, 50/20	air	—	sand	100–1000	—	—	0.40–1.70	grids above dense bed
Hanesian and Rankell (1968)	elutriation, multicomponents, batch operation	$D_c = 0.076$ $L + H = 1.37$	air	filter cloth	glass spheres	88–250	250–420	—	1.65–2.10	none
Tweddle et al. (1970)	entrainment, multicomponents, steady state operation	$D_c = 0.165$ $L/D_c = 1.3$–2.6 $H = 0.254$–2.03	air	grid	sand	104–208	—	—	0.60–0.96	screen packing above dense bed
Guha et al. (1972)	elutriation, two components, batch operation	$D_c = 0.067$ $L/D_c = 1$–2 $L + H = 0.60$	air	fixed bed of glass beads (conical shape)	sand, salt, coke, magnetite ammonium sulphate	277–359	459–677	35–80	$U_o/U_{mf} = 7$–9	none
Tanaka et al. (1972)	elutriation, two components, batch and continuous	$D_c = 0.067$ $L + H = 1.80$	air	perforated plate	glass beads, sand, stainless balls, lead balls, Neobeads	60–800	141–2300	—	1.28–2.70	none
Fournol et al. (1973)	entrainment, steady state operation	$D_c = 0.61$ $L + H = 7.92$	air	grid	FCC catalyst with size distribution	20–150	—	—	0.11–0.22	none
Harrison et al. (1974)	elutriation, two components, steady state operation	$D_c = 0.22$ $L = 0.31$ $H = 2.14$	air	—	sand	63–76	425–500	20	0.44	baffles above dense bed
Merrick and Highley (1974)	elutriation, steady state operation	$D_c = 0.90 \times 0.45$ $L = 0.61$–1.22 $L + H = 3.96$	air	—	coal ash with size distribution	0–1400	1400–3170	—	0.61–2.44	none
Nazemi et al. (1974)	entrainment, steady state operation	$D_c = 0.61$ $L + H = 7.92$	air	grid	FCC catalyst with size distribution	20–150	—	—	0.091–0.335	none
Large et al. (1976)	entrainment, steady state operation	$D_c = 0.61$ $L + H = 7.92$	air	grid	sand with size distribution	37–250	—	—	0.20–0.30	none

Investigator	Experiments	Column Diameter, D_c Bed Height, L Freeboard, H (m)	Gas	Distributor	Particles	Size of Fines to be Entrained (μm)	Size of Coarse Solids in Parent Bed (μm)	Initial Weight Fraction of Fines (%)	Superficial Gas Velocity (m/s)	Internals
Bachovchin et al. (1979)	entrainment, multicomponents, steady state operation	$D_c = 0.1524$ $L = 0.23–0.25$ $H = 0.75–4.00$	air	perforated plate	sand	22–180	Coarse: 443–1095 Medium: 22–1095	10–20	0.61–1.25	none
Geldart et al. (1979)	elutriation, multicomponents, batch operation	$D_c = 0.076$ $L = 0.35–0.45$ $L + H = 3.80$	air	filter paper covered by wire mesh	sand, shot, Alumina	38–327	150–355	5–75	0.60–3.00	none
Colakyan et al. (1979)	elutriation, multicomponents, batch operation	$D_c = 0.90 \times 0.90$ $L + H = 6.30$	air	perforated plate	sand with size distribution	37–356	356–2360	10	0.90–3.60	with or without immersed heat transfer tubes
Lin et al. (1980)	entrainment and elutriation, multicomponents, steady state operation	$D_c = 0.60 \times 0.60$ $L = 0.25$ $H = 0.63–3.27$	air	grid	sand/char with size distribution	0–125 (char)	125–419 (sand)	0.01–1	0.10–0.30	none
George & Grace (1980)	elutriation multicomponents,		air (300°K–445°K)	perforated plate	silica sand	30–90	90–272	50	0.2–1.3	with or without tube bundle

APPENDIX 2.

Published Correlations for Elutriation Rate Constant Calculation.

Yagi and Aochi (1955)

$$\frac{E_{i\infty}\, g\, d_p^2}{\mu\, (U_o - U_{ts})^2} = 0.0015\, \mathrm{Re}_t^{0.6} + 0.01\, \mathrm{Re}_t^{1.2}$$

Zenz and Weil (1958)

$$\frac{E_{i\infty}}{\rho_g U_o} = \begin{cases} 3.91 \times 10^2 \left(\dfrac{U_o^2}{g\, d_p\, \rho_s^2}\right)^{1.87}, & \dfrac{U_o^2}{g\, d_p\, \rho_s^2} \leq 581.8 \times 10^{-3} \\ 7.02 \times 10^3 \left(\dfrac{U_o^2}{g\, d_p\, \rho_s^2}\right)^{1.15}, & \dfrac{U_o^2}{g\, d_p\, \rho_s^2} \geq 581.8 \times 10^{-3} \end{cases}$$

Wen and Hashinger (1960)

$$\frac{E_{i\infty}}{\rho_g (U_o - U_{ts})} = 1.52 \times 10^{-5} \left[\frac{(U_o - U_{ts})^2}{g\, d_p}\right]^{0.5} \mathrm{Re}_t^{0.725} \left(\frac{\rho_s - \rho_g}{\rho_g}\right)^{1.15}$$

Tanaka et al. (1972)

$$\frac{E_{i\infty}}{\rho_g (U_o - U_{ts})} = 4.6 \times 10^{-2} \left[\frac{(U_o - U_{ts})^2}{g\, d_p}\right]^{0.5} \mathrm{Re}_t^{0.3} \left(\frac{\rho_s - \rho_g}{\rho_g}\right)^{0.15}$$

Merrick and Highley (1974)

$$\frac{E_{i\infty}}{\rho_g U_o} = A + 130 \exp\left[-10.4 \left(\frac{U_{ts}}{U_o}\right)^{0.5} \left(\frac{\mathrm{U_{mf}}}{\mathrm{U_o - U_{mf}}}\right)^{0.25}\right]$$

Geldart et al. (1979)

$$\frac{E_{i\infty}}{\rho_e U_o} = 23.7 \cdot \exp\left(-5.4 \frac{U_{ts}}{U_o}\right)$$

$\rho_e = \rho_g + \Sigma \rho_i$ (ρ_i: solid loading of ith size fraction in exit gas)

Colakyan et al. (1979)

$$E_{i\infty} = 33 \left(1 - \frac{U_{ts}}{U_o}\right)^2$$

Bachovchin et al. (1979)

$$E_{i\infty} = 3.35 \times 10^{-5} \left(\frac{U_o}{\sqrt{\bar{d}_p\, g}}\right)^{4.67} \left(\frac{\rho_g}{\rho_s}\right)^{1.62} \left(\frac{\mu}{\bar{d}_p}\right) \left(\frac{D_c \sqrt{X_s}}{\bar{d}_p}\right)^{1.15}$$

where

$\bar{d}_p$: average size of particle elutriated

X_s: fraction of fines at the bed surface

Lin et al. (1980)

$$\frac{E_{i\infty}}{\rho_g U_o} = 9.43 \times 10^{-4} \left(\frac{U_o^2}{g\, d_p}\right)^{1.65}$$

LITERATURE CITED

Andrews, J. M., "Kinetic Study of Fluidized Solids Entrainment," *I & EC*, **52**, No. 1, 85 (1960).

Babcock and Wilcox, "Fluidized Bed Combustion Development Facility and Commercial Utility AFBC Design Assessment," *Technical Progress Report*, EPRI (January–March, 1979).

Bachovchin, D. M., J. M. Beer, and A. F. Sarofim, "An Investigation into the Steady-State Elutriation of Fines From a Fluidized Bed," AIChE 72nd Meeting, San Francisco (1979).

Bergougnou, M. A., Personal Communication (1980).

Blyakher, I. G., and V. M. Pavlov, "Reducing Dust Entrainment from Conical Fluidized-Bed Equipment Through the Use of Support Grids," *International Chem. Eng.*, **6**, No. 1, 47 (1966).

Capes, C. E., and K. Nakamura, "Vertical Pneumatic Conveying—An Experimental Study with Particles in the Intermediate and Turbulent Flow Regimes," *Can. J. Chem. Eng.*, **51**, No. 1, 31 (1973).

Chen, T. P., and S. C. Saxena, "A Theory of Solids Projection from a Fluidized Bed Surface as a First Step in the Analysis of Entrainment Processes," Fluidization edited by J. F. Davidson and D. L. Keairns, Cambridge University Press, 151 (1978).

Colakyan, M., N. Catipovic, G. Jovanovic, and T. Fitzgerald, "Elutriation from a Large Particle Fluidized Bed with and without Immersed Heat Transfer Tubes," AIChE 72nd Meeting, San Francisco (1979).

Cranfield, R. R., and D. Geldart, "Large Particle Fluidization," *CES*, **29**, 935 (1974).

Do, H. T., J. R. Grace, and R. Clift, "Particle Ejection and Entrainment from Fluidized Beds," *Powder Technol.*, **6**, 195 (1972).

Fournol, A. B., M. A. Bergougnou, and G. G. J. Baker, "Solids Entrainment in a Large Gas Fluidized Bed," *Can. J. of Chem. Eng.*, **51**, 401 (1973).

Geldart, D., J. Cullinan, S. Georghiades, D. Gilvray, and D. J. Pope, "The Effect of Fines on Entrainment from Gas Fluidized Beds," Transction of *I. Chem. E.*, **57**, 269 (1979).

George, S. E., and J. R. Grace, "Entrainment of Particles from Aggregative Fluidized Beds," *AIChE Symposium Series*, **74**, No. 176, 67 (1978).

George, S. E., and J. R. Grace, "Entrainment of particles from a Pilot Scale Fluidized Bed," Poster Session, 3rd International Conference on Fluidization, Henniker, New Hampshire (August, 1980).

Guha, S. K., A. Kumar, and P. Sen Gupta, "Mechanism of Elutriation from Fluidized Beds," *Can. J. Chem. Eng.*, **50**, No. 5, 602 (1972).

Hanesian, D., and A. Rankell, "Elutriation from a Multisize Particle Fluidized Bed," *I & EC Fundamentals*, **7**, No. 3, 452 (1968).

Harrison, D., P. N. Aspinall, and J. Elder, "Suppression of Particle Elutriation from a Fluidized Bed," *Trans. Instr. Chem. Engrs.*, **52**, 213 (1974).

Horio, M., A. Taki, Y. S. Hsieh, and I. Muchi, "Elutriation and Particle Transport Through the Freeboard of a Gas-Solid Fluidized Bed," 3rd International Conference on Fluidization, Henniker, New Hampshire (August, 1980).

Jolley, L. T., and J. E. Stantan, "Fluidization in Beds of Coal and Coke Particles: Some Effect of Size of Particles and Viscosity, Density and Velocity of Gas," *J. Appl. Chem.*, **2**, Supplementary Issue, No. 1, 562 (1952).

Kunii, D., and O. Levenspiel, "Fluidization Engineering," John Wiley and Sons, Inc., 302 (1969).

Large, J. F., Y. Martinie, and M. A. Bergougnou, "Interpretive Model for Entrainment in a Large Gas Fluidized Bed," International Powder Bulk Solids Handling & Processing Conference (May, 1976).

Leung, L. S., R. J. Wiles, and D. H. Nicklin, "Correlation for Predicting Choking Flowrates in Vertical Pneumatic Conveying," *I&EC Process Design Develop.*, **10**, No. 2, 183 (1971).

Leva, M., "Elutriation of Fines from Fluidized Systems," *Chem. Eng. Prog.*, **47**, No. 1, 39 (1951).

Leva, M., and C. Y. Wen, "Elutriation," Chapter 14 of Fluidization edited by J. F. Davidson and D. Harrison, Academic Press, London, and N.Y. (1971).

Lewis, W. K., E. G. Gilliland, and P. M. Lang, "Entrainment from Fluidized Beds," *CEP Symposium Series*, **58**, No. 38, 65 (1962).

Lin, L., J. T. Sears, and C. Y. Wen, "Elutriation and Attrition of Char from a Large Fluidized Bed," *Powder Tech.*, **27**, 105 (1980).

Matsen, J. M., "Entrainment Research: Achievements and Opportunities," prepared for the NSF Workshop on Fluidization and Fluid-Particle Systems, Troy, New York (October, 1979).

Merrick, D., and J. Highley, "Particle Size Reduction and Elutriation in a Fluidized Bed Process," *AIChE Symposium Series*, **70**, No. 137, 366 (1974).

Mori, S., and C. Y. Wen, "Estimation of Bubble Diameter in Gaseous Fluidized Beds," *AIChE J.*, **21**, No. 1, 109 (1975).

National Coal Board (London, England), (NCB) "Reduction of Atmospheric Pollution," final report prepared for EPA (September, 1971).

Nazemi, A., M. A. Bergougnou, and G. G. J. Baker, "Dilute Phase Hold-up in a Large Gas Fluidized Bed," *AIChE Symposium Series*, **70**, No. 141, 98 (1974).

Osberg, G. L., and D. H. Charlesworth, "Elutriation in a Fluidized Bed," *Chem. Eng. Prog.*, 47, No. 11, 566 (1951).

Rowe, P. N., and B. A. Partridge, "The Interaction Between Fluids and Particles," London: The Institute of Chemical Engineers, 135 (1962).

Tanaka, I., H. Shinohara, H. Hirosue, and Y. Tanaka, "Elutriation of Fines from Fluidized Bed—Effect of Apparatus Structure," Technology Reports of the Kyushu University, **44**, No. 3, 406 (1971).

Tanaka, I., H. Shinohara, H. Hirosue, and Y. Tanaka, "Elutriation of Fines from Fluidized Bed," *J. of Chem. E. of Japan*, **5**, No. 1, 51 (1972).

Tanaka, I., and H. Shinohara, "Estimation of the Column Height of a Fluidized Bed," *International Chem. Eng.*, **18**, No. 2, 276 (1978).

Thomas, W. J., P. J. Grey, and S. B. Watkins, "Effect of Particle Size Distribution in Fluidization—Classification and Elutriation," *British Chem. Eng.*, **6**, No. 2, 176 (1961).

Tweddle, T. A., C. E. Capes, and G. L. Osberg, "Effect of Screen Packing on Entrainment from Fluidized Beds," *I & EC Process Des. Dev.*, **6**, No. 1, 85 (1970).

Wen, C. Y., "Dilute and Dense Phase Pneumatic Transport," paper in Bulk Materials Handling," **1**, edited by M. C. Hawk, Univ. of Pittsburgh, 258 (1971).

Wen, C. Y., and L. H. Chen, "A Fluidized Bed Combustor Freeboard Model," paper presented at the Sixth International Conference on Fluidized Bed Combustion, Atlanta, Georgia (April, 1980).

Wen, C. Y., and R. F. Hashinger, "Elutriation of Solid Particle From a Dense Phase Fluidized Bed," *AIChE J.*, **6**, No. 2, 220 (1960).

Wen, C. Y. and W. S. O'Brien, "Pneumatic Conveying and Transporting," Chapter 3 in Gas-Solids Handling in the Process Industries, edited by J. M. Marchello, and A. Gomezplata, Marcel Dekker, Inc., 89 (1976).

Wen, C. Y. and Y. H. Yu, "A Generalized Method for Predicting the Minimum Fluidization Velocity," *AIChE J.*, **12**, No. 3, 610 (1966).

Yagi, S., and T. Aochi, "Elutriation of Particles from a Batch Fluidized Bed," paper presented at the Society of Chemical Engineers (Japan), Spring Meeting (1955).

Yang, W. C., "A Mathematical Definition of Choking Phenomenon and a Mathematical Model for Predicting Choking Velocity and Choking Voidage," *AIChE J.*, **21**, No. 5, 1013 (1975).

Yates, J. G., and P. N. Rowe, "A Model for Chemical Reaction in the Freeboard Region Above a Fluidized Bed," Trans. *Instn. Chem. Engrs.*, **55**, 137 (1977).

Zenz, F. A., and N. A. Weil, "A Theoretical-Emperical Approach to the Mechanism of Particle Entrainment from Fluidized Beds," *AIChE J.*, **4**, No. 4, 472 (1958).

Zenz, F. A., Personal Communication (1980).

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